Reply to the Comment on "Theorem on the proportionality of inertial and gravitational masses in classical mechanics"

Andrew E. Chubykalo and Stoyan J. Vlaev
Escuela de Física, Universidad Autónoma de Zacatecas
Apartado Postal C-580 Zacatecas 98068, ZAC., México
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In a preceding Comment, the author declares that we claim that the ratio of inertial mass to gravitational mass can be derived *ex nihilo* and that our paper was published by mistake. In this "Reply" we dispute the point of view of the author.

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In the preceding Comment [1], B. Jancovici tries to argue why in his opinion our proof makes no sense: "*Reading the paper, one realized that it contains irrelevant calculations ... aiming to prove that η is a constant for one given body: how could the ratio of two constants be something else?"* The point is that after reading our paper, anyone can realize that we *do not use* a postulate of classical mechanics that mass of a body is a *constant*. In other words, we prove that ratio η of inertial m_i to gravitational m_g is a *constant* without postulating that masses m_i and m_g are absolute constants. To be more specific, let us compare postulates used by a generally accepted classical mechanics (GACM) with the postulates used in our proof.

Postulates of GACM:

(a_0) *any* body with non-zero inertial mass possesses also non-zero gravitational mass;

(a) (the Equivalence principle) the inertial mass of the body is proportional to the gravitational mass of the same body and a constant of the proportionality is the same for *all* bodies;

(b) the masses of bodies (inertial and gravitational) are absolute constants (invariant);

(c) masses obey the principle of additivity.

Note: In GACM the claim that

$$\text{from} \quad (a_0) + (b) + (c) \Rightarrow (a) \tag{1}$$

is not quite obvious from a theoretical point of view: for example, let us consider two *concrete* bodies with the different masses m_1 and m_2. Their *constant* masses $m_{i1}, m_{i2}, m_{g1}, m_{g2}$, obviously, must obey the relations

$$m_{i1}/m_{g1} = \eta_1 \qquad \text{and} \qquad m_{i2}/m_{g2} = \eta_2 \tag{2}$$

but, generally speaking, it is not obvious that

$$\eta_1 = \eta_2. \tag{3}$$

So in GACM one considers Eq.(3) as an experimental fact. However, after one applies our arguments (see Eqs. (15)-(21) in [7]) one can prove the validity of the claim (a) (or Eq.(3)).

An important remark about the postulate (b)**:** in his famous book [2] E.Mach convincingly shows (analyzing Newton's well-known experiment with the "revolving pail") that all experimentally verifiable equations of Newtonian classical mechanics do not change if one supposes that inertial mass of a body is not an absolute constant (invariant) and, generally speaking, it can depend on the location of a body in space. So many scientists (see, e.g., [3-5]), following Mach's ideas, expect that the principle of the proportionality of m_i and m_g may not be valid for classical mechanics. That is why when constructing general relativity, Einstein started with the Mach principle, but had to reject it thereafter (e.g.,see [6]) because of its disagreement (as Einstein believed) with the Equivalence principle. And that is why a proof of the postulate (a) also in the framework of Mach's ideas is of great interest.

Our postulates:

(a_0) *any* body with non-zero inertial mass possesses also non-zero gravitational mass;

(b_1) the masses of bodies (inertial and gravitational) do not depend explicitly on time but they *can* depend on their location in space;

(c_1) both inertial and gravitational masses obey the principle of additivity.

It is obvious that our postulates are weaker than the former ones. One can see that we do not conserve the point (a) as a postulate, bat what's more, our postulate (b_1) sufficiently differs from the postulate (b). The validity of the point (a) from the postulates $(a_0), (b_1), (c_1)$ also (compare with Eq.(1)) is not obvious, because m_i and m_g may depend on location *in different ways*. This is so, because m_i and m_g have a *different* origin in classical mechanics.

In our paper we proved that from our postulates (a_0), (b_1) and (c_1) one infers the claim (a). Actually, we proved that if according to the Mach principle, the inertial mass

of a body can change from point to point in space, then the gravitational mass of the same body *must* also change by the same law, i.e. m_i and m_g are linear dependent (proportional) one-to-one functions. In other words, we show that even in the framework of the Mach principle the proportionality of inertial and gravitational masses *must* take place.

In the last paragraph of the Comment, the author advances his most serious critical remark. However, at this point he misses an implicit but very important factor:

Indeed, after one applies our arguments (see [7], Eqs. (6)-(11)), and provided that one neglects electromagnetic radiation, we can formally obtain an expression

$$m_i = \text{const } q_m, \tag{4}$$

(where q_m is a charge of the *given* body m) for a *concrete* body with a *given* inertial mass and a *given* velocity. But we *cannot* apply the subsequent arguments (see [7], Eqs.(15)-(21)) in order to "prove" that *all* particles have the same charge-to-mass ratio. The point is that in our speculations we implicitly use the postulate a_0, namely, "*any* body with non-zero inertial mass possesses also non-zero gravitational mass". In other words, after Eq. (11) (in [7]) we *can* claim that for bodies having the same mass, their m_i satisfy Eq.(11) from [7]. But we *cannot* claim *this* in the discussion *concerning* "inertial mass-charges" relation. In the latter case one should postulate *that any body with non-zero inertial mass possesses also non-zero charge*. It is obvious that such a "postulate" does not follow from the experiment.

As a finishing remark, let us note that reading the Abstract of the "Comment", a reader might come to the conclusion that our paper was published by mistake, after being rejected by the Editorial Board. In fact, the paper was published with minor modifications following a positive report by an anonymous referee selected by the Board. However, we recognize that the critical remarks contained in the "Comment" are partly justified: we should have explicitly defined our postulates.

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[7] Chubykalo A and Vlaev S 1998 Eur. J. Phys. Vol.19 1